INFLUENCE OF THICKNESS AND WALL THERMAL CONDUCTIVITY ON HEAT TRANSFER IN LAMINAR NATURAL CONVECTION OF AIR IN A CUBICAL CAVITY

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Results are presented for a numerical investigation of the influence of finite thermal conductivity and wall thickness on heat transfer in a vertical cavity of rectangular cross section. Theoretical formulas are obtained for determining the total heat flux.

The flow and heat transfer in a vertical cavity of rectangular cross section, under natural convection conditions (NC), have been studied by many authors [1-6]. As a rule, the temperature conditions at the body—liquid interface (the temperature or the heat flux) are assumed to be known beforehand. However, this approach is not always satisfactory [7]. The assignment of surface temperature under steady-state heat transfer is valid only in the case of infinite body thermal conductivity, whereas in the real physical situation the walls have finite thermal conductivity and thickness. Under unsteady heat transfer the law for variation of surface temperature with time is not known beforehand. Therefore, in the design and construction of technical installations, the structure and the liquid interact appreciably, it is desirable to treat the thermal problem as related, i.e., to seek a simultaneous solution of the equations of convection of the liquid and the equations for thermal conductivity in the body, with the temperatures and heat fluxes, which are not known beforehand, [13] equal at the phase interface surface. The criterion for the interaction of the temperature field on the body and of the liquid washing it under NC conditions is the complex dimensionless group Br, the Brown number, which is a quantity proportional to the ratio of the thermal resistance of the washed wall to the thermal resistance of the liquid [8, 11]:

$$\mathrm{Br} = \frac{k_f \delta}{k_s x} \mathrm{Pr}^{1/4} \mathrm{Gr}_x^{1/4}, \quad 10^2 \leqslant \mathrm{GrPr} \leqslant 2 \cdot 10^7,$$

where x is the distance along the wall in the flow direction; δ is the wall thickness; and $k_f(k_s)$ is the thermal conductivity of the liquid (the wall). It is assumed that in the region Br > 0.02 the problem must be solved in the joint formulation [8].

1. In this paper we use numerical modeling to investigate two-dimensional laminar NC in a rectangular cavity filled with air, with solid impermeable walls.

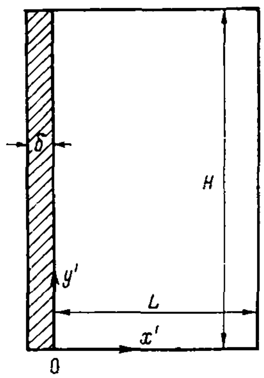

Fig. 1. Physical model.

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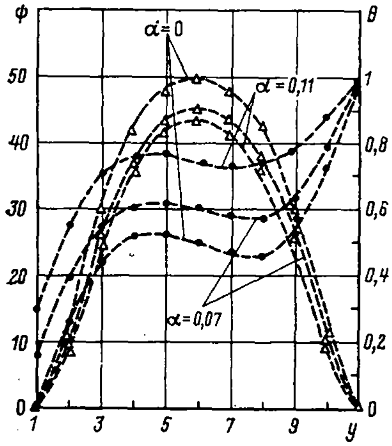

Fig. 2

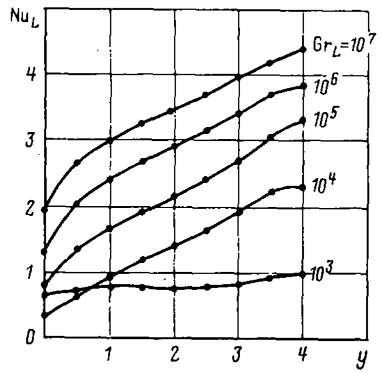

Fig. 3

Fig. 2. The temperature and velocity profiles as a function of the coupling parameter α $[\theta|_{x=1}=1, \mathrm{Gr_L}=10^5,\ y = H/(2L),\ H/L = 6]$. The triangles show ψ (the stream function), and the points show θ (the temperature).

Fig. 3. The local Nusselt number as a function of y for $\alpha = 0.14$; $H/L = 4$; $\theta|_{x=1} = 1$.

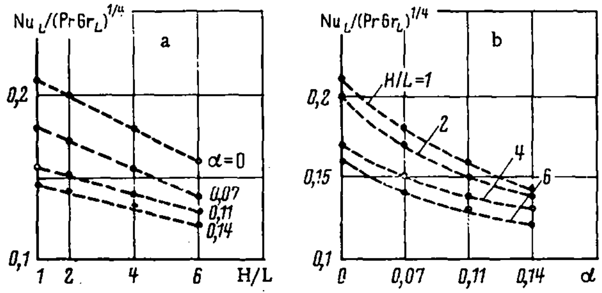

Fig. 4. Heat transfer as a function of α (H/L) $[\mathrm{Gr_L} = 10^5,\ y = H/(2L),\ \theta|_{x=1} = 1]$.

The physical model of the problem is illustrated in Fig. 1. The left vertical wall is a flat plate of finite thickness. The top and bottom boundaries of the cavity $[y' = 0,\ y' = H]$ are thermally insulated, and the vertical boundaries $[x' = -\delta,\ x' = L]$ are kept at constant and different temperatures $(T_1 \neq T_0)$.

If we restrict attention to the case where the transverse thermal conductivity (in the direction x') in the plate $[-\delta \le x' \le 0,\ 0 \le y' \le H]$ is considerably greater than the longitudinal conductivity (in the direction y'), then the conditions for the heat fluxes to be equal at the interface surface $(x' = 0)$ may be reduced to the boundary condition

$$k_f \frac{\partial T(0,\ y')}{\partial x'} = \frac{k_s}{\delta}[T(0,\ y') - T_0].$$

Therefore, the boundary problem considered (in dimensionless variables) can be formulated as follows:

$$\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y} = \frac{1}{\mathrm{Pr}}\left(\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2}\right),$$

$$\frac{\partial\psi}{\partial y}\frac{\partial\Omega}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial\Omega}{\partial y} = \frac{\partial^2\Omega}{\partial x^2} + \frac{\partial^2\Omega}{\partial y^2} + \mathrm{Gr}_L\frac{\partial\theta}{\partial x},$$

$$\Omega = -\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2}\right),$$

$$\psi = \frac{\partial\psi}{\partial y} = 0,\quad \frac{\partial\theta}{\partial y} = 0;\quad 0 \leqslant x \leqslant 1;\quad y = 0,\ \frac{H}{L}, \tag{1}$$

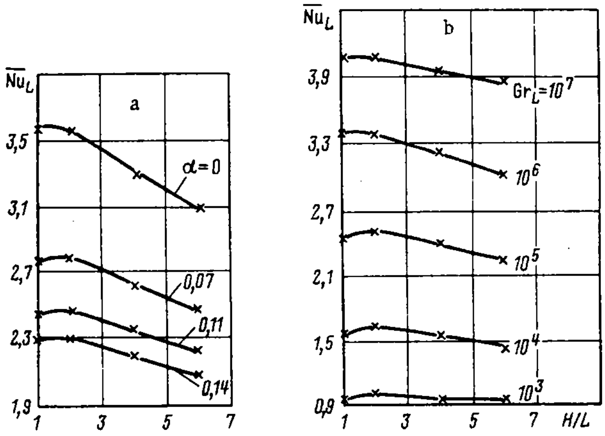

Fig. 5. The average Nusselt number as a function of H/L [a) $\theta|_{x=1} = 1$, $Gr_L = 10^5$; b) $\theta|_{x=1} = 1$, $\alpha = 0.11$].

$$\psi = \frac{\partial\psi}{\partial x} = 0, \quad \alpha\frac{\partial\theta}{\partial x} = \theta; \quad x = 0, \quad 0 \leqslant y \leqslant \frac{H}{L},$$

$$\psi = \frac{\partial\psi}{\partial x} = 0, \quad \theta = \begin{cases} +1, & \text{if} \quad T_1 > T_0, \\ -1, & \text{if} \quad T_0 > T_1, \end{cases} \quad x = 1, \quad 0 \leqslant y \leqslant \frac{H}{L}. \tag{1}$$

Here as the scales for distance, the stream function, and the vorticity we choose the cavity width L, the kinematic viscosity ν and the quantity (L^2/ν), respectively. The dimensionless temperature is $\theta = (T-T_0)/\Delta T$, $\Delta T = T_{max}-T_{min}$. In system (1) we have the dimensionless groups, the Prandtl number $Pr = \nu/a$, and the Grashof number $Gr_L = g\beta\Delta T L^3/\nu^2$, and also the coupling parameter $\alpha = (k_f/k_s) \times (\delta/L)$, which describes the influence of the finite conductivity (k_f/k_s) of the solid and liquid phases and the geometric factor (δ/L) on the flow and the heat transfer in the rectangular cavity heated from the side.

The complete solution of the coupled problem (1) can be defined by the functional dependence

$$\left.\begin{matrix}\theta \\ \psi\end{matrix}\right\} = f\left(x, y, Gr_L, \alpha, \frac{H}{L}\right). \tag{2}$$

2. System (1) was solved numerically by the finite-differences method. A monotonic conservative difference scheme of second-order accuracy [6] was used. A steady-state solution was found by the iterative Zeidel process. Relaxation parameters, for which an optimal value was determined by experimental calculations, were used to accelerate the convergence of iterations in the vorticity and stream function equation. The calculations were carried out in a uniform mesh with a spatial step of $h = 1/20$.

3. The following matters were clarified from an analysis of the numerical data obtained for the values $Gr_L = 10^3$-10^7, $\alpha = 0$; 0.07; 0.11; 0.14, H/L = 1; 2; 4; 6; 10 at a fixed Prandtl number of Pr = 0.72.

The influence of Grashof number Gr_L (at fixed α) on the thermoconvective processes in the cavity, elucidated by solving the coupled problem (1), is close to the results obtained earlier [3, 5] in the solution of the analogous problem in the uncoupled form. Increase in Grashof number leads to intensification of convection, the generation of a constant vertical temperature gradient and subsequent change of thermal conditions in the cavity. The regime close to a heat-conduction condition ($Gr_L \leqslant 10^4$) is replaced by a boundary layer regime ($Gr_L \sim 10^5$). The isotherms are almost horizontal in the center of the cavity. Further increase in Grashof number ($Gr_L \geqslant 10^6$) leads to curvature of the isotherms in the center and the appearance of secondary circulatory flows described in detail in [9].

We now consider the influence of the coupling of the heat transfer on the intensity of convective motion and on the temperature field structure. With increase in the coupling parameter α for a given temperature drop ($\Delta T = T_1-T_0$), the convective intensity decreases, e.g., for $\alpha \sim 0.1$, by 20% in comparison with $\alpha = 0$.

TABLE 1. Theoretical Formulas for the Average Nusselt Number

α	$H/L=1$	$H/L=2, 4, 6$
0	$\overline{Nu}_L=0{,}225(Gr_L)^{0,243}$ (3′)	$\overline{Nu}_L=0{,}226(Gr_L)^{0,245}(H/L)^{-0,155}$ (3″)
0,07	$\overline{Nu}_L=0{,}283(Gr_L)^{0,193}$ (4′)	$\overline{Nu}_L=0{,}267(Gr_L)^{0,212}(H/L)^{-0,125}$ (4″)
0,14	$\overline{Nu}_L=0{,}337(Gr_L)^{0,16}$ (5′)	$\overline{Nu}_L=0{,}332(Gr_L)^{0,175}(H/L)^{-0,109}$ (5″)

The isotherm field undergoes considerable changes here. A decrease in the intensity of convective motion with increase of α leads to an increase in the liquid temperature. A typical example of the influence of α ($\alpha = 0$–0.11; $Gr_L = 10^5$; $H/L = 6$; $y = H/(2L)$; $\theta|_{x=1} = 1$) is shown in Fig. 2.

Analysis of the distribution of local heat transfer $Nu_L = -\partial\theta/\partial x|_{x=0}$ on the surface $x = 0$ for $\theta|_{x=1} = 1$ ($T_1 > T_0$), $H/L = 2; 4; 6; 10$, $Gr_L = 10^3$–10^7 and with α fixed shows that: for small Grashof number ($Gr_L \sim 10^3$) the value of Nu_L almost does not vary with height of the wall; for values $Gr_L = 10^4$–10^7, $H/(4L) \le y \le H/(7L)$ there is a variation $Nu_L = 0.45y + f(Gr_L)$. An increase in Gr_L is accompanied by an increase in Nu_L over practically the entire height of the wall. On the cold wall [$x = 0$, $0 \le y \le H/L$; $\theta|_{x=1} = 1$] the quantity Nu_L reaches a maximum near the top of the wall, since here the temperature gradients are greatest because of flow out from the wall [$x = 1$, $0 \le y \le H/L$] of liquid which has been heated more; the quantity Nu_L depends slightly on the ratio H/L. A typical distribution of local heat flux is shown in Fig. 3.

It can be shown by the methods of similarity theory [10] that, in the general case,

$$Nu_L = \varphi(\alpha,\ H/L)(PrGr_L)^{1/4}.$$

Calculations made for $x = 0$, $y = H/(2L)$, $Gr_L = 10^5$, $Pr = 0.72$, $\theta|_{x=1} = 1$ show that: the dependence of the heat transfer $\varphi(\alpha,\ H/L) = Nu_L/(PrGr_L)^{1/4}$ on H/L, with α as a parameter, is linear (Fig. 4a); if we take H/L as a parameter, a monotonic decrease of $\varphi(\alpha,\ H/L)$ with increase of α is observed (Fig. 4b).

We shall describe the heat transfer to the cavity in terms of an average Nusselt number $\overline{Nu}_L = -\frac{L}{H}\cdot\int_0^{H/L}[\partial\theta/\partial x]|_{x=0}\,dy$. The decrease in intensity of convection, observed with increase in the coupling parameter α, leads to a reduction in the average Nusselt number $\overline{Nu}_L$. Here the average Nusselt number $\overline{Nu}_L$ depends substantially on the cavity geometry and on the Gr_L number. Typical curves showing $\overline{Nu}_L$ as a function of (H/L), Gr_L, and α are shown in Fig. 5.

The relation between $\log\overline{Nu}_L$ and $\log Gr_L$ is almost linear, and for $10^3 \le Gr_L \le 10^6$, $H/L = 2, 4, 6$, and α fixed one obtains parallel straight lines. Thus, one can construct a formula of the type $\overline{Nu}_L = A(Gr_L)^B(H/L)^C$. For the case $H/L = 1.10^3 \le Gr_L \le 10^6$, $\alpha = 0; 0.07; 0.14$ the least-squares method was used to obtain interpolation relations of the type $\overline{Nu}_L = A(Gr_L)^B$. The results are shown in Table 1. Comparison of the calculated values of $\overline{Nu}_L$ with the Eqs. (3′) and (3″), obtained here for $\alpha = 0$, with the values calculated using the Elder formula [2]

$$\overline{Nu}_L = 0.231\,(G_L)^{0.25},\quad Pr = 0.733,\ H/L = 1,\ Gr_L Pr > 4\cdot 10^3 \tag{3}$$

and the Yakob formula [12]

$$\overline{Nu}_L = 0.18\,(Gr_L)^{0.25}(H/L)^{-0.111},\quad H/L = 2;\ 10,\ 10^3 \leqslant Gr_L \leqslant 10^6 \tag{4}$$

show that values of $\overline{Nu}_L$ from Eq. (3′) fall about 8-10% lower, while values of $\overline{Nu}_L$ from Eq. (3″) fall about 9-11% above the values of $\overline{Nu}_L$ calculated from Eqs. (3) and (4), respectively.

Calculations using Eqs. (3′)-(5″) show that, for all values of Gr_L, with increase of the coupling parameter α, the average Nusselt number $\overline{Nu}_L$ decreases. For example, if the value of $\overline{Nu}_L$, for $Gr_L = 10^5$, from Eq. (3′), and $\alpha = 0$ ($Br = 0$), is taken as 100%, then the values of $\overline{Nu}_L$ from Eqs. (4′) and (5′), corresponding to $\alpha = 0.07$ ($Br = 1.14$), $\alpha = 0.14$ ($Br = 2.27$), are 70 and 58%, respectively.

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